An active control logic based on modal approach for vibration reduction through the eigenstructure assignement

Mattia Serra
Department of Mechanics
Politecnico di Milano
Milano, Italy
Email: mattia.serra@mail.polimi.it

Ferruccio Resta
Department of Mechanics
Politecnico di Milano
Milano, Italy
Email: ferruccio.resta@polimi.it

Francesco Ripamonti
Department of Mechanics
Politecnico di Milano, Milano, Italy
Email: francesco.ripamonti@polimi.it
Telephone: (+39) 0223998480
Fax: (+39) 0223998492

***Abstract*—This paper presents a new control technique for vibration reduction based on modal approach and named Dependent Modal Space Control (DMSC). It is an active control logic that allows to assign the closed loop poles and the corresponding mode shapes of a system in a discrete number of degree of freedom depending on the amount of the sensors and actuators available. In order to perform the eigenstructure assignment, modal sensors and actuators are necessary unless both control and observation spillover are minimized.**

I. Introduction

In recent years, vibration control has become increasingly relevant. Consequently, a lot of effort is spent on its study. This is due to the fact that light and flexible structures generally have low damping ratio, especially on the first modes, which in many application are also the most excited. Thus, vibrations cause fatigue problems or simply downgrade the performance, exceeding the acceptable thresholds in precision applications. Among the different vibration control strategies, one of the most used is the modal control introduced by Balas [1] and Meirovitch [2] between the 1970s and 1980s. A few years after the introduction of modal control, Meirovitch proposed the Independent Modal Space Control (IMSC), which uses a modal filter to estimate the modal states needed by the control law [3]. Starting from the original IMSC, many improvements have been developed in order to increase the performance and reduce the effects of control and observation spillover [4, 5]. To achieve this results Baz and Poh [6] and then Fang, Li and Jeary [7] developed the Modified Independent Modal Space Control. A further improvement of the MIMSC was carried out by Singh and Agarwal [8]; in their strategy, the energy of different modes is checked at specific intervals of time and the highest part of control effort is always directed towards the dominant modes. Kim and Inman [9, 10, 11] developed a Sliding mode observer in order to eliminate the observation spillover improving the performance of the IMSC and ensuring higher robustness to the control method.
It is important to remark that all these methods make it possible to independently modify frequencies and damping ratios of the controlled modes without affecting the system mode shapes through a diagonal gain matrix in principal coordinates [12, 13, 14, 15].
On the contrary, this paper presents a strategy to place not only the desired controlled eigenvalues but also the eigenvectors (as a linear combination of the uncontrolled ones) exploiting the non-diagonal entries of the control gain matrix. In the case of distributed sensor-actuators, thus in absence of spillover, it is possible to create virtual nodes in desired locations of the controlled modes with great advantages in many applications thanks to the eigenstructure assignment.
To demonstrate the method performances, in the present paper, a clamped beam [16] is controlled with three modal actuators and three modal sensors. Numerical simulations are carried out and the differences between the IMSC and DMSC are presented.

II. Dependent Modal Space Control

Consider the equation of motion of a generic mechanical system

$$[\mathbf{M}]\,\ddot{\underline{\mathbf{z}}} + [\mathbf{R}]\,\dot{\underline{\mathbf{z}}} + [\mathbf{K}]\,\underline{\mathbf{z}} = [\mathbf{\Lambda_{F_c}}]^{\mathbf{T}}\underline{\mathbf{F}}_{\mathbf{c}} + [\mathbf{\Lambda_{F_d}}]^{\mathbf{T}}\underline{\mathbf{F}}_{\mathbf{d}} \quad (1)$$

where $[\mathbf{M}]$, $[\mathbf{R}]$, $[\mathbf{K}]$ are the inertial, damping and stiffness matrices, the vector $\underline{\mathbf{z}}$ contains the independent variables in physical coordinates, the matrices $[\mathbf{\Lambda_{F_c}}]^{\mathbf{T}}$ and $[\mathbf{\Lambda_{F_d}}]^{\mathbf{T}}$ link the displacement of force application points with the independent variables and $[\mathbf{F_c}]$, $[\mathbf{F_d}]$ are respectively the control and the disturbance forces acting on the system. In order to implement the control logic and write the closed loop matrices, the disturbance force will be omitted in the following.
The change in principal coordinates

$$\underline{\mathbf{z}} = [\mathbf{\Phi}]\underline{\mathbf{q}}_{\mathbf{n}} \quad (2)$$

is performed through a transformation matrix $[\mathbf{\Phi}] \in \mathbf{R}^{\mathbf{nxn}}$, containing the eigenvectors of $[\mathbf{M}]^{-\mathbf{1}}[\mathbf{K}]$. If the structural damping satisfies the Rayleigh assumption, i.e. $[\mathbf{R}] = \alpha[\mathbf{M}] + \beta[\mathbf{K}]$, the transformation matrix $[\mathbf{\Phi}]$ is able to diagonalize the matrix differential equation such that each mode evolves independently from the others. The system equations

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partitioned in two subsystems named respectively with index m (modeled) and nm (not modeled) become

$$\begin{cases} [\mathbf{M_{qm}}]\underline{\ddot{\mathbf{q}}}_{\mathbf{m}} + [\mathbf{R_{qm}}]\underline{\dot{\mathbf{q}}}_{\mathbf{m}} + [\mathbf{K_{qm}}]\underline{\mathbf{q}}_{\mathbf{m}} = [\mathbf{B_{qcm}}]\underline{\mathbf{F}}_{\mathbf{c}} \\ [\mathbf{M_{qnm}}]\underline{\ddot{\mathbf{q}}}_{\mathbf{nm}} + [\mathbf{R_{qnm}}]\underline{\dot{\mathbf{q}}}_{\mathbf{nm}} + [\mathbf{K_{qnm}}]\underline{\mathbf{q}}_{\mathbf{nm}} = [\mathbf{B_{qcnm}}]\underline{\mathbf{F}}_{\mathbf{c}} \end{cases} \tag{3}$$

Considering the modeled subsystem, it is possible to write its equations in state space form

$$\underline{\dot{\mathbf{x}}}_{\mathbf{qm}} = [\mathbf{A_{qm}}]\underline{\mathbf{x}}_{\mathbf{qm}} + [\mathbf{B_{xqcm}}]\underline{\mathbf{F}}_{\mathbf{c}} + [\mathbf{B_{xqdm}}]\underline{\mathbf{F}}_{\mathbf{d}} \tag{4}$$

and computing the control force

$$\underline{\mathbf{F}}_{\mathbf{c}} = [\mathbf{B_{qcm}}]^{-\mathbf{1}}[[\mathbf{G_v}]\ [\mathbf{G_p}]]\underline{\mathbf{x}}_{\mathbf{qm}} \tag{5}$$

the closed loop matrix results

$$[\mathbf{A_{qmcl}}] = [[\mathbf{A_{qm}}] + [\mathbf{B_{xqcm}}][\mathbf{B_{qcm}}]^{-\mathbf{1}}\underbrace{[[\mathbf{G_v}][\mathbf{G_p}]]}_{[\mathbf{G}]} \tag{6}$$

Assigning the $[\mathbf{A_{qmcl}}]$ eigenvalues allows to select the desired non-dimensional damping and frequencies of the controlled modes. The $[\mathbf{A_{qmcl}}]$ eigenvectors imposition instead, determines a combination of the open loop uncoupled mode shapes. A block scheme representation of the controlled system highlighting the presence of modal sensors and actuators is shown in figure 1.

In order to compute the control matrix $[\mathbf{G}]$, it is possible to

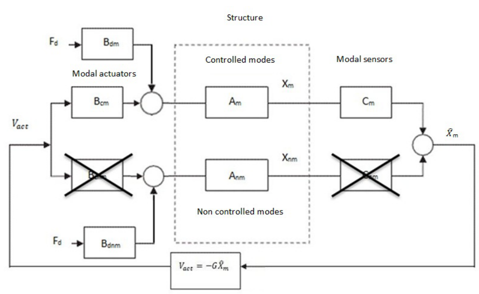

Figure 1. Block scheme of the controlled system

start from the closed loop dynamic matrix $[\mathbf{A_{qmcl}}]$ that has the desired eigenstructure

$$[\mathbf{A_{qmcl}}] = [\mathbf{T}][\mathbf{\Lambda}][\mathbf{T}]^{-\mathbf{1}} \tag{7}$$

where $[\mathbf{T}]$ and $[\mathbf{\Lambda}]$ are respectively the matrix containing the eigenvectors and the eigenvalues of $[\mathbf{A_{qmcl}}]$. Once $[\mathbf{A_{qmcl}}]$ is determined, it is possible through (6) to compute $[\mathbf{G}]$.

Consider that three modes have to be controlled (m=3) and denote three independent variables in physical coordinates as (z_{ia}, z_{ib}, z_{ic}) for the i-th mode shape. By using the change of coordinates in (2), it is possible to write the relation

$$\begin{bmatrix} z_{ia} \\ z_{ib} \\ z_{ic} \end{bmatrix} = \begin{bmatrix} \Phi_{1a} \\ \Phi_{1b} \\ \Phi_{1c} \end{bmatrix} w_{i1} + \begin{bmatrix} \Phi_{2a} \\ \Phi_{2b} \\ \Phi_{2c} \end{bmatrix} w_{i2} + \begin{bmatrix} \Phi_{3a} \\ \Phi_{3b} \\ \Phi_{3c} \end{bmatrix} w_{i3} \tag{8}$$

where a linear combination of the uncontrolled mode shapes in the selected physical coordinates are suitably weighted in order to obtain the desired closed loop mode shape. Writing the same equations also for the remaining two mode shapes, it is possible to obtain

$$\begin{bmatrix} z_{1a} & z_{2a} & z_{3a} \\ z_{1b} & z_{2b} & z_{3b} \\ z_{1c} & z_{2c} & z_{3c} \end{bmatrix} = \begin{bmatrix} \Phi_{1a} & \Phi_{2a} & \Phi_{3a} \\ \Phi_{1b} & \Phi_{2b} & \Phi_{3b} \\ \Phi_{1c} & \Phi_{2c} & \Phi_{3c} \end{bmatrix} \begin{bmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \\ w_{13} & w_{23} & w_{33} \end{bmatrix} \tag{9}$$

The columns of matrix $[\mathbf{W}]$ consist of the eigenvectors of the reduced order model in principal coordinates imposed by the control law. The dimension of matrix $[\mathbf{T}]$ is double with respect matrix $[\mathbf{W}]$ due to the difference between the mechanical and the state space formulation and they are related through the corresponding eigenvalues. Note that the IMSC is a particular case of the DMSC where the $[\mathbf{W}]$ coefficients matrix in equation (9) is an identity, leaving the mode shapes unaltered.

The conditions required to ensure the existence of a unique solution for $[\mathbf{W}]$ have to be taken into account in the selection of the z_{ij} values. In equation (9), the matrix $[\mathbf{\Phi_{abc}}]$, as a subset of $[\mathbf{\Phi}]$, is always invertible due to the orthogonal propriety of the mode shapes. Thus, to ensure $[\mathbf{W}]$ to be non-singular, the matrix $[\mathbf{Z}]$ has to be non-singular.

Note that, considering the uncontrolled mode shapes as real due to low structural stiffness, the proposed logic combines them linearly through real coefficients; thus $[\mathbf{W}]$ is imposed with real entries.

Even if the exact modal coordinates were available for control (by using for example a modal sensor), in the closed loop system, considering all the modes, the effect of control spillover couples the non-controlled modes with the controlled ones.

Thus the nm modes contribute to the motion of the system in the frequencies corresponding to the modeled m modes, preventing the impositions in the desired DoFs. For these reasons the proposed DMSC, imposing the closed loop eigenvectors, requires modal sensors and modal actuators in order to be effective. Figure 2 shows the difference between structure of the closed loop matrices related to the IMSC and the DMSC control logics. The red cross underlines the absence of control spillover thanks to the modal actuator while in the green box it is visible how the controlled modes evolves independently in the IMSC and dependently in the DMSC due to the imposition of the mode shapes.

III. Numerical Results

A numerical model describing the dynamics of a clamped aluminium beam in the vertical plane (figure 3) has been obtained through the FEM approach and will be used to test the proposed logic. The system has 21 nodes (one constrained) with 3 DoFs per node (axial and transversal displacement and bending rotation) resulting in a total of $n = 60$ DoFs. The characteristics of the clamped beam are shown in table I.

The frequencies and the non-dimensional damping of the

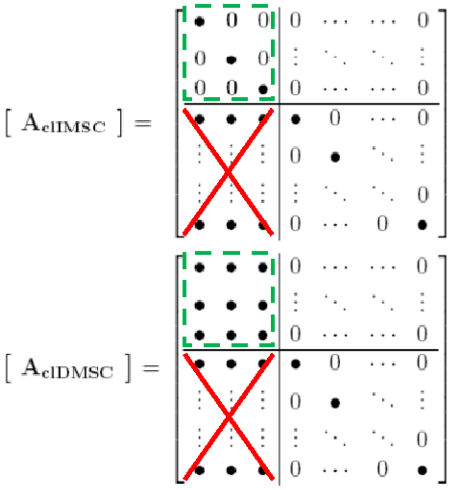

Figure 2. Structures of the closed loop matrices considering the modeled and non modeled modes

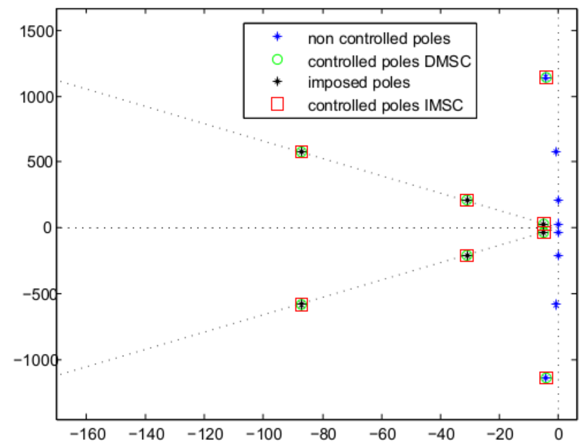

Figure 4. Open loop poles and closed loop poles for IMSC and DMSC

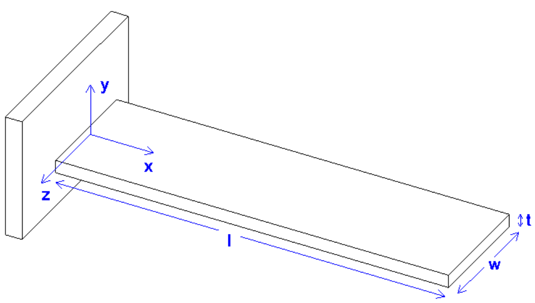

Figure 3. The cantilever beam: main dimensions and reference system

Table I
THE CANTILEVER BEAM MAIN CHARACTERISTICS

l (m)	w (m)	t (mm)	E (MPa)	m (kg/m)
1	0.04	6.1	75000	0.754

system first three modes are shown in table II.

Table II
THE CANTILEVER BEAM MODEL PARAMETERS (ω AND ξ) FOR THE FIRST 3 MODES

mode n°	1	2	3
f[Hz]	5.3	33	92.4
ξ%	0.42	0.13	0.2

The controlled poles are selected imposing a damping ratio $\xi = 0.15$ and leaving the frequencies ω unaltered with respect the non controlled system.
This is represented in figure 4 where the black dotted line indicates poles that have $\xi = 0.15$. A comparison between the classical IMSC and the DMSC is presented in order to show the advantages of the proposed method.
The system is excited by a vertical disturbance force F_d on the tip of the beam and the displacements $Y_{a,b,c}$, respectively at 0.2 [m], 0.5 [m] and 0.8 [m] from the clamp, are reported. The closed loop poles selected are the same for the two methods and the only difference between them consists in the mode shapes imposition. In the following, numerical results will show how it is possible to create virtual nodes in desired points of the structure for the first $m = 3$ controlled modes. Figure 5 shows the uncontrolled modes versus the controlled ones; the markers indicate the imposed values of $z_{i;a,b,c}$.

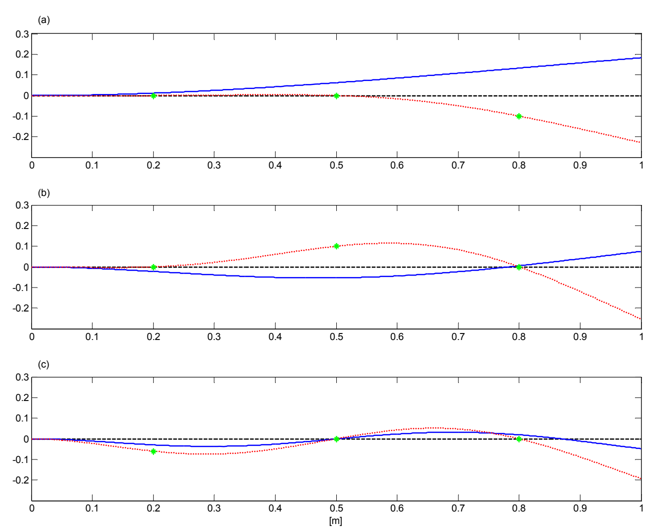

Figure 5. DMSC: original (continuous line) and imposed (dotted line) mode shapes (in star the markers); mode 1 at 5.27 Hz (a), mode 2 at 33.02 Hz (b), mode 3 at 92.46 Hz (c)

In order to ensure the existence of $[\mathbf{W}]$, as shown in equation (9), it is not possible to impose a node in the same point for

all the m controlled modes, otherwise $[\mathbf{Z}]$ would result to be singular. Figure 6, 7, 8 and 9 present respectively the FRF between F_d and the three outputs $Y_{a,b,c}$ and the three control forces applied by the modal actuators.
In each FRF between the disturbance and the three outputs, it can be noted the presence of a single resonance peak for the first three eigenfrequencies, due to the creation of virtual nodes. Furthermore, the absence of spillover in correspondence of the fourth resonance frequency is justified, for both the logics, by the application of the modal sensors and actuators. Figure 9 instead, shows that the mode shapes imposition leads to an increase of control forces with respect to the classical IMSC.

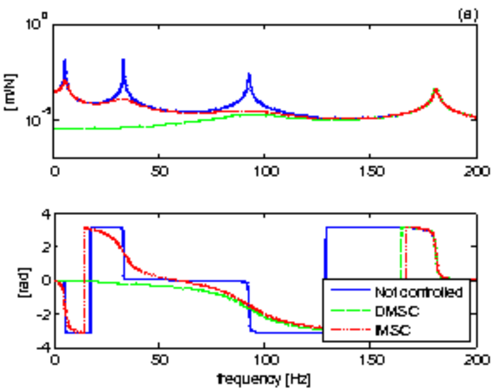

Figure 6. DMSC: FRF between the disturbance F_d and the output Y_a

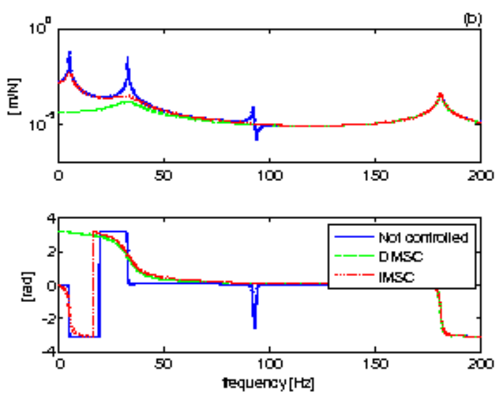

Figure 7. DMSC: FRF between the disturbance F_d and the output Y_b

A time domain analysis is carried out too. In particular, the impulse response is shown in figure 10 to emphasize the performances of the DMSC compared with the IMSC in a wide frequency range. Evident improvements in terms of vibration reduction are achieved by the DMSC with respect to the IMSC with the same controlled poles.

IV. Conclusion

In this paper a control strategy based on modal approach and called Dependent Modal Space Control (DMSC) has been

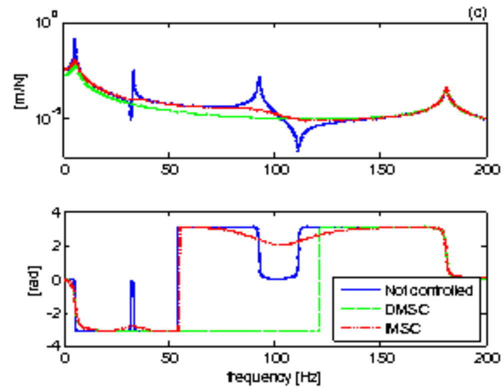

Figure 8. DMSC: FRF between the disturbance F_d and the output Y_c

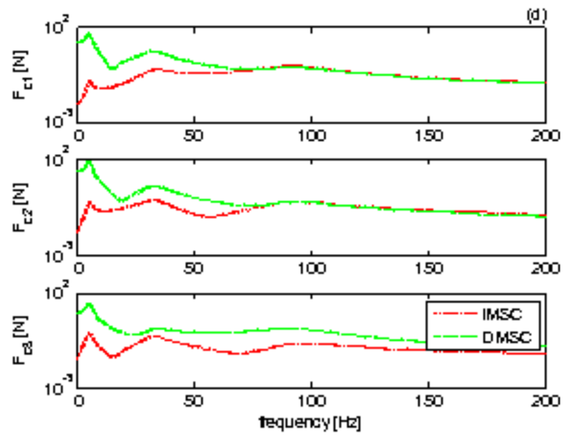

Figure 9. DMSC: FRF between the disturbance F_d and the three modal control forces F_c

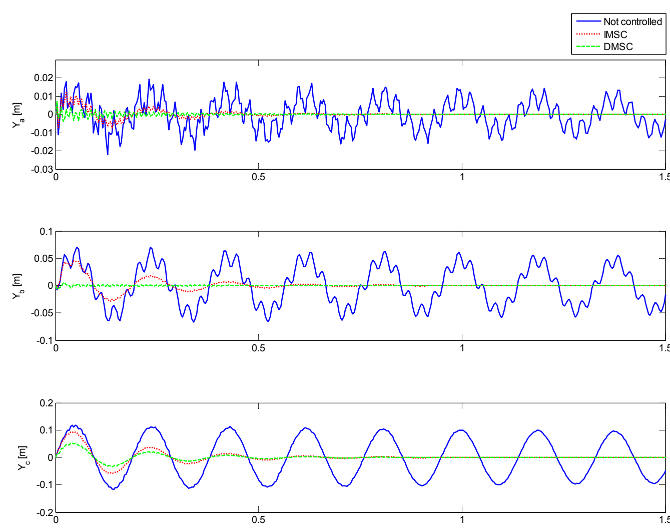

Figure 10. DMSC eigenvector assignment: response to impulse disturbance time history in the measurement points for the non controlled, IMSC and DMSC

presented. Besides the assignment of the controlled poles, the imposition of the controlled mode shapes in a discrete number of points allows to obtain virtual nodes. A main difference between this method and the IMSC is the possibility to compute a control law that is focused on the vibrations reduction in desired points with consequent advantages in many engineering applications. However, due to the modification of the mode shapes, the control forces required by the DMSC are generally higher as compared to the IMSC.

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